ASC2018: Asymptotic Statistics and Computations

Graduate School of Mathematical Sciences, The University of Tokyo (Komaba Campus)
Room 002, February 5, 2018
http://www.ms.u-tokyo.ac.jp/access_e/index_e.html

This workshop aims at exchanges of the state-of-the-art in a wide range of asymptotic statistics, computational statistics, and statistical modeling from theoretical, methodological, and implementation points of view.

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http://www.sigmath.es.osaka-u.ac.jp/statmodel

Program (Ver. February, 1)

10:25-10:30 Opening

10:30-11:15 Yuta Koike (University of Tokyo)
Multi-scale analysis of lead-lag relationships in high-frequency financial markets

11:15-12:00 Teppei Ogihara (Institute of Statistical Mathematics)
Statistical inference for integrated diffusion processes

12:00-13:30 Luncheon party (Lever son Verre)

13:30-14:15 Yuliya Mishura (Taras Shevchenko National University of Kyiv)
Fractional irregularity

14:15-15:00 Yukai Yang (Uppsala University)
Specification, estimation and evaluation of vector smooth transition autoregressive models with applications

15:00-15:30 Break

15:30-16:15 Hiroki Masuda (Kyushu University)
Efficient estimation of stable Lévy process

16:15-17:00 Nakahiro Yoshida (University of Tokyo)
Asymptotic expansion of Skorohod integrals

17:00-17:05 Closing

Multi-scale analysis of lead-lag relationships in high-frequency financial markets

Yuta Koike
University of Tokyo

We propose a novel estimation procedure for scale-by-scale lead-lag relationships of financial assets observed at high-frequency in a non-synchronous manner. The proposed estimation procedure does not require any interpolation processing of original datasets and is applicable to those with highest time resolution available. Consistency of the proposed estimators is shown under the continuous-time framework that has been developed in our previous work [1]. An empirical application shows promising results of the proposed approach.

This is a joint work with Prof. Takaki Hayashi of Keio University.

References

[1] Hayashi, T. & Koike, Y. (2016). Wavelet-based methods for high-frequency lead-lag analysis. Working paper. Available at arXiv: https://arxiv.org/abs/1612.01232.

Statistical inference for integrated diffusion processes

Teppei Ogihara
Institute of Statistical Mathematics

We study statistical inference for integrated diffusion processes and consider asymptotic properties of this model in a high-frequency limit. This model arises when we observe a process after passage though an electric filter, and is also related to modeling of the stochastic volatility in finance. Gloter and Gobet (2008) studied this model and showed the local asymptotic mixed normality (LAMN) when the diffusion process is one-dimensional. The LAMN property is important in asymptotic statistical theory and enables us to discuss asymptotic efficiency of estimators. We extend their results of the LAMN property to multi-dimensional diffusion processes which may have a feedback from the integral process. Then we can apply these results to a Langevin equation which is a model for molecular motion. This is joint work with Prof. M. Fukasawa in Osaka University.

Fractional irregularity

Yuliya Mishura

Taras Shevchenko National University of Kyiv

We consider three problems concerning Gaussian processes belonging to quasihelix:
–asymptotic behavior of maximal functionals;
–representation theorems involving integrals w.r.t. such processes;
–statistical parameter estimation.

Specification, estimation and evaluation of vector smooth transition autoregressive models with applications

Yukai Yang
Uppsala University

We consider a nonlinear vector model called the logistic vector smooth transition autoregressive model. The bivariate single-transition vector smooth transition regression model of Camacho (2004) is generalised to a multivariate and multi-transition one. A modelling strategy consisting of specification, including testing linearity, estimation and evaluation of these models is constructed. Nonlinear least squares estimation of the parameters of the model is discussed. Evaluation by misspecification tests is carried out using tests derived in a companion paper. The use of the modelling strategy is illustrated by two applications. In the first one, the dynamic relationship between the US gasoline price and consumption is studied and possible asymmetries in it considered. The second application consists of modelling two well known Icelandic riverflow series, previously considered by many hydrologists and time series analysts.

Efficient estimation of stable Lévy process

Hiroki Masuda (Kyushu University)

Objective and background.

Let $(X_t)_{t\geq 0}$ be a β-stable Lévy process with a drift μ and symmetric jumps:

$$X_t = \mu t + \sigma J_t,$$

where the driving Lévy process is characterized by $\mathbb{E}(e^{iuJ_1}) = \exp(-|u|^\beta)$. Assume that X is observed at $t_j = t_j^n = jh_n$ in such a way that

$$h_n \to 0 \quad \text{and} \quad \liminf_n nh_n > 0,$$

where h denotes a sampling stepsize; for example, $h_n = T/n$ for a fixed terminal sampling time T. We are concerned here with asymptotically efficient estimation of the three-dimensional unknown parameter

$$\theta := (\beta, \sigma, \mu) \in (0,2) \times (0,\infty) \times \mathbb{R}.$$

Indeed, any proper form of efficient result has been missing in the literature: It is known that the joint estimation of the scale σ and stable index β under any *diagonal-matrix* norming leads to the *constantly singular* Fisher information matrix ([1], [3, 4]), making the conventional minimax results due to Hájek-Le Cam not directly applicable.

Results in brief.

We will present the following:

(1) The best possible rate of convergence and the minimal asymptotic covariance matrix of a regular estimator of θ;
(2) An easy-to-compute and numerically stable asymptotically optimal estimator.

For the first one, we show the local asymptotic normality property with a *non-diagonal* norming rate and *non-singular* Fisher information matrix, and also the asymptotic normality of a sequence of local maxima $\hat{\theta}_n$ of the likelihood. This enables us to precisely describe efficiency in this statistical experiment; in particular, it is shown that a normalized estimator of the form $\varphi_n(\theta)^{-1}(\hat{\theta}_n - \theta)$ weakly converges to a non-degenerate normal distribution only for a class of *non-diagonal* $\varphi_n(\theta)$.

The second one is of practical concern, for, although the exact likelihood function can be numerically computed [5], the associated likelihood function involves the local scaling factor "$h^{-1/\beta}$". This will make repetitive numerical optimization for a maximum-likelihood estimate much more time-consuming compared with the i.i.d. model setup, while on the other hand the proposed one-step strategy only requires numerical integration, which is already available in R.

The presented results are based on the joint work with Alexandre Brouste [2].

References

[1] Aït-Sahalia, Y. and Jacod, J. (2008) Fisher's information for discretely sampled Lévy processes, Econometrica, 76(4), 727–761.

[2] Brouste, A. and Masuda, H. (2017), Efficient estimation of stable Lévy process with symmetric jumps, submitted.

[3] Masuda, H. (2009) Joint estimation of discretely observed stable Lévy processes with symmetric Lévy density. Journal of the Japan Statistical Society 39, no.1, 49–75.

[4] Masuda, H. (2015) *Parametric Estimation of Lévy Processes*, in *Lévy Matters IV*, Lecture Notes inMathematics 2128, Springer.

[5] Matsui, M. and Takemura, A. (2006) Some improvements in numerical evaluation of symmetric stable density and its derivatives. Comm. Statist. Theory Methods 35, no. 1-3, 149–172.

Asymptotic expansion of Skorohod integrals

Nakahiro Yoshida
University of Tokyo

Asymptotic expansion of the distribution of a perturbation Z_n of a Skorohod integral jointly with a reference variable X_n is derived. We introduce a second-order interpolation formula in frequency domain to expand a characteristic functional and combine it with the scheme developed in the martingale expansion. The second-order interpolation and Fourier inversion give asymptotic expansion of the expectation $E[f(Z_n, X_n)]$ for differentiable functions f and also measurable functions f. In the latter case, the interpolation method connects the two non-degeneracies of variables for finite n and ∞. Random symbols are used for expressing the asymptotic expansion formula. Quasi tangent, quasi torsion and modified quasi torsion are introduced in this paper. We identify these random symbols for a certain quadratic form of a fractional Brownian motion and for a quadratic from of a fractional Brownian motion with random weights. For a quadratic form of a Brownian motion with random weights, we observe that our formula reproduces the formula originally obtained by the martingale expansion.